

Introduction To Euclid's Geometry || CLASS - 9th

Euclidean geometry is the study of plane and solid figures on the basis of axioms and theorems employed by the Greek mathematician Euclid.

Exercise 5.1

Q1 Which of the following statements are true and which are false? Give reason for your answer.

i) Only one line can pass through a single point.

Ans: → False. Infinitely many lines can pass through a point.

ii) There are an infinite number of lines which pass through two distinct points.

Ans: → False. Through two distinct points only one line can pass.

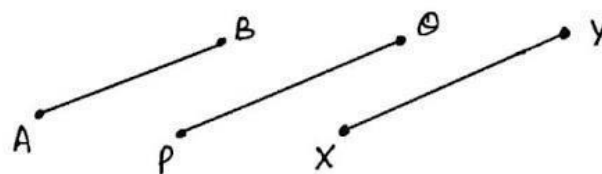
iii) A terminated line can be produced infinitely on both the sides.

Ans: → True. A terminated line or line segment can be produced indefinitely on both sides to give a line.

iv) If two circles are equal, then their radii are equal.

Ans: → True. Two circles of equal area will have the same radius from the relation $\text{area} = \pi r^2$.

v) In the figure, if $AB = PO$ and $PO = XY$ then $AB = XY$.



Ans: → True. From the axiom that if two things are separately equal to a third thing, then they are equal to each other.

Q2: → Give a definition for each of the following terms.

Are there other terms that need to be defined first? What are they and how might you define them?

- i) parallel lines ii) perpendicular lines. iii) Line segment
iv) radius of circle v) Square

Ans: → Parallel Lines: → Two straight lines which have no point in common are said to be parallel to each other

Perpendicular Lines: → If one among two parallel lines is turned by 90° the two lines become perpendicular to each other.

Line Segment: → A line with two end point is a line segment

Radius of a circle: → The line segment with one end point at the centre and the other at any point on the circle

Square: → A quadrilateral with all sides equal and all angles right angle is square.

Q3 Consider two postulates given below:

- i) Given any two distinct point A and B, there exist a third point C which is between A and B
- ii) There exist at least three points that are not on the same line.

Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid's postulates Explain?

Ans: → In postulate (i) in between A and B remains an undefined term which appeals to our geometric intuition

The postulates are consistent. They do not contradict each other. Both of these postulates do not follow from Euclid's postulates. However they follow from the axiom given below.

Given two points there is a unique line that passes through them.

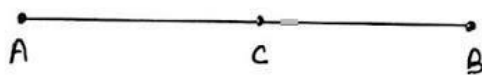
i) Let $A \longrightarrow B$ AB be a straight line.

There are an infinite number of points composing this line. Choose any except the two end-points A and B . This point lies between A and B .

ii) If there are only two points, they can always be connected by a straight line. Therefore, there have to be at least three points for one of them not to fall on the straight line between the other two.

Q4 If a point C lies between two points A and B such that $AC = \frac{1}{2} AB$. Explain by drawing the figure.

Ans : \rightarrow



$$AC = CB$$

Also $AC + CA = BC + AC$ [equal are added to equals]

$BC + AC$ coincides with AB

$$\Rightarrow 2AC = AB$$

$$\Rightarrow AC = \frac{AB}{2}$$

Q5 In Question 4, point C is called a mid point of line segment AB . Prove that every line segment has one and only one mid point.

Ans: → Let there be two such mid point C and D

Then from above theorem

$$AC = \frac{1}{2} AB$$

$$\text{and } AD = \frac{1}{2} AB$$

$$\therefore AC = AD$$

But this is possible only if D coincides with C. Therefore C is the unique mid-point. Proved

Q6: → In fig. If $AC = BD$, then prove that $AB = CD$

Ans: → Given $AC = BD$

To prove $AB = CD$

$$AC = AB + BC$$

$$BD = BC + CD$$

$$\text{As } AC = BD \text{ (Given)}$$

$$AB + BC = BC + CD$$

$$\therefore AB = CD \text{ Proved}$$

Q7: → Why is Axiom 5, in the list of Euclid's axioms, considered a universal truth? (Note that the question is not about the fifth postulate)

Ans: → Whole is always greater than its part

This is a 'universal truth' because part is included in the whole and therefore can never be greater than the whole in magnitude.

Exercise 5.2

Q1 How would you rewrite Euclid's fifth postulate so that it would be easier to understand?

Ans: → When two lines are cut by a third line, such that the sum of interior angles is less than 180° on one side then first two lines intersect on the same side.

Q2 Does Euclid's fifth postulate imply the existence of parallel lines? Explain.

Ans: → It may be argued that Euclid's fifth postulate recognises the existence of parallel lines. If sum of interior angle is 180° on both sides of transversal then the lines will not intersect on any side. So if two lines never intersect then they are parallel.